I. PROBLEM SESSION 5

A. Problem 5.1

a)Recall the spectra of acustical and optical phonons, explain the qualitative difference.
b)How does the density of states look? How does it depend on dimensions of the sample?
c)How does the heat capacity depend on temperature.
d)Explain differences in the contribution of optical and acoustical phonons to the heat capacity.
e)Explain the specific features of the Einstein and Debye models.

B. Problem 5.2

a)From the dispersion relation of a monoatomic linear lattice of N atoms with nearest neighbour interactions, show that the density of modes is

$$D(\omega) = \frac{2N}{\pi\sqrt{{\omega_m}^2 - \omega^2}} \tag{1}$$

reduces to the continuum elastic wave equation where ω_m is the maximum frequency.
b)Suppose that an optical phonon branch has the form $\omega(K) = \omega_0 - AK^2$ near $K = 0$, in three dimensions. Show that

$$D(\omega) = \frac{L}{2\pi}^3 \frac{2\pi\sqrt{\omega_0 - \omega}}{A^{\frac{3}{2}}} \tag{2}$$

for $\omega < \omega_0$, and $D(\omega > \omega_0) = 0$

C. Problem 5.3

Heat capacity of a layered lattice:
Consider a dielectric crystal made up of layers of atoms with rigid coupling between the layers so that the motion of atoms is restricted to the plane of the layer. Show that the phonon heat capacity in the Debye approximation is proportional to T^2 in the low temperature limit.